domain near the pole of the unit sphere. The wave field is constructed still more simply at the unit sphere poles themselves, as (2.8) shows. The deductions made are also carried over without difficulty to the case of a rotating uniformly stratified fluid. For this case the FFIWO is obtained in [3, 4].

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INSTABILITY WAVE EXCITATION BY A LOCALIZED VIBRATOR IN THE BOUNDARY LAYER

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Modern methods of determining the critical Reynolds numbers for the laminar-turbulent boundary layer transition are based on computations of the linear stage of instability wave development (Tollmien—Schlichting waves), whose initial amplitudes are determined empirically [1, 2]. An analysis of the possible excitation mechanism for Tollmien—Schlichting (T—S) waves by external perturbations is needed to construct closed algorithms to compute the development of these waves. On the other hand, the problem of boundary layer susceptibility to external effects is closely associated with questions of flow laminarization on aircraft vehicles.

It is well known that spatially localized, external perturbations that are periodic in time excite T—S waves effectively [1-3]. Under real flight conditions and in wind tunnel tests the source of such perturbations might be the vibration of the surface being streamlined. Experiments in [4, 5] indicate the close relationship between the origination of instability and the characteristics of model vibration.

A theoretical analysis of the perturbations excited by vibrations of a surface being streamlined in the boundary layer was developed in [6-11]. The first mathematical model related to T—S wave generation by a localized vibrator was constructed by Gaster [6]. A vibrator in sub- and supersonic boundary layers was examined in [7-9]. The analysis was performed within the framework of a three-layer asymptotic model under the assumption that the characteristic scales of the problem correspond to the neighborhood of the lower branch of the neutral curve. Asymptotic expressions are obtained for the longwave pressure perturbations excited in the neighborhood of the vibrating section of the surface. In particular, the amplitude of the damped T—S wave is determined. However, the problem of excitation of growing instability waves emerged beyond the framework of the mathematical model used. In this connection, a postulate was proposed in [10] that permits determination of the amplitude

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of an exponentially increasing T–S wave. The need for the postulate vanishes if a vibrator is considered that starts to vibrate at a certain initial time t = 0 [6], or the problem is solved in large length scales with the flow in the boundary layer not being parallel taken into account (see below).

The resonance mode of T–S wave excitation by a vibration wave traveling over the streamlined surface is examined in [11]. The analysis is performed by using an eigenfunction expansion of the solution of the linearized Navier–Stokes equations. Here the flow not being parallel in a real boundary layer was taken into account. The high efficiency of resonance excitation of T–S waves was detected.

To confirm the theoretical results, an experimental investigation was performed on the perturbations excited by a localized vibrator in the boundary layer on a flat plate [12].

In this paper instability wave excitation by a localized vibrator is analyzed on the basis of the results in [11]. Theory is compared with experiment. The amplitudes of the instability waves being excited are computed for the Mach numbers M = 0.2-0.8.

1. Let us consider the flow in a two-dimensional boundary layer. For simplicity in the exposition, we consider the flow incompressible. The nonessential distinctions that occur when taking account of compressibility of the medium are indicated below. Let x be the downstream distance from the model leading edge along the streamlined surface, and y the distance along the normal therefrom. A vibrator oscillating at the frequency ω (Fig. 1) is installed at a distance L from the leading edge. We select the characteristic scales: the distance $X \sim L$ along the x axis, $(\nu_\infty X/U_\infty)^{1/2}$ along the y axis, where ν_∞, U_∞ are the kinematic viscosity coefficient and the free-stream velocity. The time is measured in the units $(\nu_\infty X/U_\infty^3)^{1/2}$, the pressure in the units $\rho_\infty U_\infty^2$, where ρ_∞ is the density.

We assume that the main flow is weakly inhomogeneous in the x direction with longitudinal U and normal V* velocity components satisfying the relations

$$U = U(x, y),\ V^* = \varepsilon V(x, y),\ \varepsilon = R^{-1} = (\nu_\infty / U_\infty X)^{1/2} \ll 1.$$

Let the surface vibrations be described by the equation

$$y_w(s, t) = \mathrm{Real}\left[a f(s)\, e^{-i\omega t}\right],\quad s = \varepsilon^{-1}(x - x_*),\quad f(s) = O(1),$$

where $x_* = L/X$ is the coordinate of the vibrator center, and f(s) is the vibrator shape localized in x in the scale $\sim\varepsilon L$. It is assumed that the amplitude of the vibrations is much less than the thickness of the viscous near-wall layer $a \ll (\omega R)^{-1/2}$. In this case the perturbations excited by the vibrator in the boundary layer are described by the linearized Navier–Stokes equations [7-10].

We introduce the vector function Q(x, y, t): Q_1, Q_2, Q_3 are perturbations of the x velocity component, the pressure, and the y velocity component, respectively, $Q_4 = \partial Q_1/\partial y - \varepsilon \partial Q_3/\partial x$. For fixed-frequency perturbations

$$\mathbf{Q}(x, y, t) = a\,\mathrm{Real}[\mathbf{A}(x, y) e^{-i\omega t}],$$

where **A**(x, y) is the complex amplitude satisfying the linearized Navier–Stokes equations in which a Fourier transformation in the time has been executed [11-13]:

$$\partial \mathbf{A}/\partial y - H_1 \mathbf{A} = \varepsilon H_2 \partial \mathbf{A}/\partial x + \varepsilon H_3 \mathbf{A}. \tag{1.1}$$

The explicit form of the matrices H_1 and H_2 is given in [11]. The matrix-operator H_3 contains the terms $\partial U/\partial x$, V, $\partial V/\partial y$, related to the main flow not being parallel.

The following boundary conditions are satisfied for an impermeable surface

$$\begin{gathered}\mathrm{Real}[a A_1(x, y_w) e^{-i\omega t}] + U(x, y_w) = 0,\\ \mathrm{Real}[a A_3(x, y_w) e^{-i\omega t}] + V^*(x, y_w) - \partial y_w/\partial t = 0.\end{gathered} \tag{1.2}$$

Expanding (1.2) in a series in the neighborhood of y = 0, we have to $O(a^2) + O(\varepsilon a)$ accuracy

$$A_1(x, 0) = -U'_w f(s),\quad A_3(x, 0) = -i\omega f(s),\quad U'_w = \frac{\partial U}{\partial y}(x, 0). \tag{1.3}$$

As $y \to \infty$, boundedness is assumed for the perturbations

$$|\mathbf{A}| < \infty,\ y \to \infty. \tag{1.4}$$

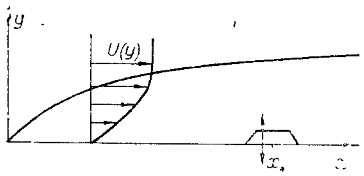

Fig. 1

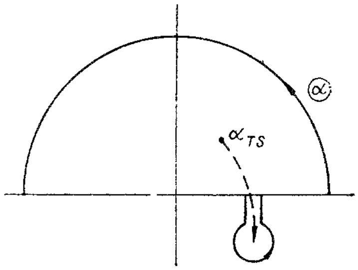

Fig. 2

We give the initial data

$$\mathbf{A}(x_0,\ y) = \mathbf{A}_0(y) \tag{1.5}$$

in the section $x = x_0$ sufficiently far upstream of the vibrator.

In general, the mixed problem (1.1), (1.3)-(1.5) is incorrect. It is necessary that A_0 allow construction of a solution with a finite growth index downstream [11, 14].

2. We represent the vibrator shape f(s) as a Fourier integral in the wave numbers

$$f = \int\limits_{-\infty}^{+\infty} \rho(\alpha_v) \exp\left[i\varepsilon^{-1}\alpha_v(x - x_*)\right] d\alpha_v. \tag{2.1}$$

The shape $f = \rho(\alpha_v)\exp[i\varepsilon^{-1}\alpha_v(x - x_*)]$ of a vibrating surface corresponding to one harmonic from the integral (2.1) is examined in [11]. It is shown that in this case the solution $A(x, y, \alpha_v)$ is represented in the form of an expansion in a biorthogonal system of eigenfunctions $\{A_\alpha, B_\alpha\}$, supplemented by the inhomogeneous solution for $y = 0$

$$\mathbf{A}(x, y, \alpha_v) = \sum_\alpha{}' c_\alpha(x)\, \mathbf{A}_\alpha(x, y)\, e^{\varphi_\alpha(x)} + \mathbf{A}_v(x, y)\, e^{\varphi_v(x)},$$

$$\varphi_\alpha(x) = i\int\limits_{x_0}^{x} \varepsilon^{-1}\alpha dx, \quad \varphi_v(x) = i\int\limits_{x_*}^{x} \varepsilon^{-1}\alpha_v dx.$$

Analysis of the spectrum and fundamental properties of the eigenfunctions is carried out in [11, 14]. The orthogonality relationships

$$\langle H_2\mathbf{A}_\alpha,\ \mathbf{B}_\beta\rangle = \Delta_{\alpha\beta},$$

$$\langle H_2\mathbf{A}, \mathbf{B}\rangle = \int\limits_0^\infty (H_2\mathbf{A}, \mathbf{B})\, dy, \quad (H_2\mathbf{A},\ \mathbf{B}) = \sum_{i,j=1}^{4} H_2^{ij} A_j \bar{\mathbf{B}}_i$$

are satisfied, where $\Delta_{\alpha\beta}$ is the Kronecker symbol when at least one of the eigenvalues belongs to a discrete spectrum, $\Delta_{\alpha\beta} = \delta(\alpha - \beta)$ is the delta function if α, β refer to the continuous spectrum (the upper bar denotes the complex conjugate).

The vector function $\mathbf{A}_v(x, y)$ is a solution of the system

$$\frac{\partial \mathbf{A}_v}{\partial y} - H_1\mathbf{A}_v = i\alpha_v H_2 \mathbf{A}_v, \quad A_{v1} = -U'_w\rho(\alpha_v), \quad A_{v3} = -i\omega\rho(\alpha_v), \quad y = 0; \ |\mathbf{A}_v| \to 0, \ y \to \infty$$

and describes the perturbation induced in a locally homogeneous flow by a vibration wave with wave number α_v. The following relationship holds [11]

$$\langle H_2\mathbf{A}_v,\ \mathbf{B}_\alpha\rangle i(\alpha_v - \alpha) + (\mathbf{A}_v,\ \mathbf{B}_\alpha)_{y=0} = 0. \tag{2.2}$$

Let the coordinate x_* of the center of the vibrator agree with the point $x_{1.s.}$ of loss of stability of the T–S wave (a generalization is given below for a vibrator located to the right or left of $x_{1.s.}$). Then a resonance mode of T–S wave excitation $\alpha_v = \alpha_{v*} = \alpha_{TS}(x_*)$ is possible at the point x_*, where $\alpha_{TS}(x_*)$ is the wave number for the T–S wave. The perturbation amplitude $A(x, y, \alpha_{v*})$ has an asymptotic for $\varepsilon \to 0$, $x - x_* \gg \sqrt{\varepsilon x_*}$ [11]:

$$\mathbf{A}(x, y, \alpha_{v*}) = D(x, \varepsilon)\,\mathbf{A}_{TS}(x, y),$$

$$D(x, \varepsilon) = i\int_{x_0}^{x} \frac{\alpha_v - \alpha_{TS}}{\varepsilon(x - x_*)} \exp\left[i\int_{x_0}^{x} \frac{\alpha_v - \alpha_{TS}}{\varepsilon}\,dx\right] dx \exp\left[-\int_{x_0}^{x_*}\left(i\frac{\alpha_{TS}}{\varepsilon} + W_{TS,TS}\right)dx\right], \tag{2.3}$$

where A_{TS} is the eigenfunction of the T–S wave

$$W_{TS,TS} = -\langle H_2 \partial \mathbf{A}_{TS}/\partial x,\ \mathbf{B}_{TS}\rangle - \langle H_3 \mathbf{A}_{TS},\ \mathbf{B}_{TS}\rangle.$$

The small parameter ε in the eikonal $\varphi = i\int_{x_0}^{x} \varepsilon^{-1}(\alpha_v - \alpha_{TS})\,dx$ permits determination of the asymptotic of the integral in (2.3) by the saddle-point method. The amplitude Φ_{TS} of the T–S wave excited at resonance is determined by the neighborhood of the saddle point x_* and has the form [11]

$$\mathbf{\Phi}_{TS}(x, y, \alpha_{v*}) = a\rho(\alpha_{v*})\, g\mathbf{A}_{TS} e^{F_{TS}},$$

$$g = \sqrt{\frac{2\pi}{\varepsilon\left(\frac{d\alpha_{TS}}{dx}\right)_*}}\,(U'_w B_{TS1} + i\omega B_{TS3})_{*,y=0}\, e^{i\Phi} + O(\varepsilon^{1/2}), \tag{2.4}$$

$$F_{TS} = \int_{x_*}^{x}\left(i\frac{\alpha_{TS}}{\varepsilon} + W_{TS,TS}\right)dx,$$

where Φ is a real constant defined by the selection of the branch of the root in (2.4). The quantities with the asterisk (*) subscript are evaluated at the point x_*.

For a small detuning from resonance $\Delta\alpha = \alpha_v - \alpha_{TS}(x_*)$, the saddle point z_* defined by the equation $\alpha_v - \alpha_{TS} = 0$ will be complex

$$z_* = x_* + \frac{\Delta\alpha}{\left(\frac{d\alpha_{TS}}{dx}\right)_*} + O(\Delta\alpha^2).$$

Let us continue the total solution $A(x, y, \alpha_v)$ in a small domain of the complex z plane including $\sqrt{\varepsilon}$, the neighborhood of the saddle point z_* (it is assumed that $\mathbf{A}(z, y, \alpha_v)$ is analytic in this domain). We expand $\mathbf{A}(z, y, \alpha_v)$ in the neighborhood of z_* exactly as was done for x_* in [11], and we evaluate $D(z, \varepsilon)$ by the saddle-point method. We consequently obtain that the amplitude of the excited T–S wave is

$$\mathbf{\Phi}_{TS}(x, y, \alpha_v) = a\rho(\alpha_v)\, g \exp\left[\frac{i}{2\varepsilon}\frac{(\alpha_v - \alpha_{v*})^2}{\left(\frac{d\alpha_{TS}}{dx}\right)_*}\right] e^{F_{TS}}\mathbf{A}_{TS}. \tag{2.5}$$

Integrating $\Phi_{TS}(x, y, \alpha_v)$ over all α_v belonging to the resonance interaction domain $|\alpha_v - \alpha_{v*}| \sim \sqrt{\varepsilon}\alpha_{v*}$, for $x - x_* >> \sqrt{\varepsilon}x_*$ we have an asymptotic estimate for the total amplitude of the instability wave excited by a localized vibrator

$$\mathbf{\Phi}_{TS\Sigma} = 2\pi a\rho(\alpha_{v*})\,(U'_w B_{TS1} + i\omega B_{TS3})_{*,y=0}\,\mathbf{A}_{TS} e^{F_{TS}}, \tag{2.6}$$

where

$$\rho(\alpha_{v*}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(s)\, e^{-i\alpha_{v*}s}\,ds.$$

If the vibrator is shifted from the point of loss of stability, α_{v*} will be complex. We continue $\mathbf{A}(x, y, \alpha_v)$ into the domain of the complex α_{v*} plane including $\sqrt{\varepsilon}$, the neighborhood of the point $\alpha_{v*} = \alpha_{TS}(x_*)$. It is easy to show that the relationships (2.4) and (2.5) do not change. To evaluate the total amplitude we deform the contour of integration in α_v, so that it would pass through the saddle point α_{v*} along the line of steepest descent. We consequently obtain that as in the case of real α_{v*}, $\Phi_{TS\Sigma}$ is determined by the expression (2.6).

Therefore, generation of a T–S wave is localized in the flow interval $|x - x_*| \sim \sqrt{\varepsilon}x_*$ and occurs in a narrow band of vibrator wave numbers $|\alpha_v - \alpha_{v*}| \sim \sqrt{\varepsilon}\alpha_{v*}$ concentrated around the resonance value $\alpha_{v*} = \alpha_{TS}(x_*)$. The instability wave being excited for $x - x_* >> \sqrt{\varepsilon}x_*$

is developed according to the solitary wave laws. Since the excitation is localized in x, the relationship (2.6) is valid for boundary layers on bodies of sufficiently smooth shape with the characteristic scale $L \gg (\nu_\infty X/U_\infty)^{1/2}$.

3. Taking account of the weak inhomogeneity in the main flow is the essence of the analysis in Sec. 2. The problem of perturbations excited by a vibrator in a plane-parallel boundary layer was solved by asymptotic methods in [9]. As a comparison with the results in [9], we examine this problem by using the expansion in a biorthogonal system of eigenfunctions $\{\mathbf{A}_\alpha, \mathbf{B}_\alpha\}$.

Let the vibrator be on the bottom of a plane-parallel boundary layer corresponding to the Reynolds number $\mathrm{Re} = (U_\infty X/\nu_\infty)^{1/2}$. We locate the center of the vibrator at the point $x_* = 0$. As in [9], we require that $|\mathbf{A}| \to 0$, $y \to \infty$, $x \to -\infty$. In a certain section $x = x_1$ downstream from the vibrating section of the surface, the perturbation amplitude $\mathbf{A}_1 = \mathbf{A}(x_1, y)$ satisfies the homogeneous boundary conditions on the wall $A_1(0) = A_3(0)$ and the boundedness condition $|\mathbf{A}| < \infty$, $y \to \infty$. Then $\mathbf{A}(x, y)$ can be expanded in a complete system of eigenfunctions $\{\mathbf{A}_\alpha, \mathbf{B}_\alpha\}$ in the domain $x > x_1 > 0$ [14]:

$$\mathbf{A}(x, y) = \sum_\nu \langle H_2\mathbf{A}_1, \mathbf{B}_{\alpha_\nu}\rangle \exp\left[i\varepsilon^{-1}\alpha_\nu (x - x_1)\right] \mathbf{A}_{\alpha_\nu}(y) + \sum_j \int_0^\infty \langle H_2\mathbf{A}_1, \mathbf{B}_{\alpha_j}\rangle \exp\left[i\varepsilon^{-1}\alpha_j (x - x_1)\right] \mathbf{A}_{\alpha_j}(y)\, dk,$$

where $\sum_\nu$ is the summation over the discrete spectrum, and $\sum_j$ is the sum of integrals over the continuous spectrum branches $\alpha_j = \alpha_j(k)$, $0 < k < \infty$.

Let us expand $\mathbf{A}(x, y)$ in a Fourier integral in the wave numbers α_v:

$$\mathbf{A}(x, y) = \int_{-\infty}^{+\infty} \mathbf{A}_v(y, \alpha_v) \exp\left[i\varepsilon^{-1}\alpha_v x\right] d\alpha_v.$$

Taking account of (2.1), we have from (2.2)

$$\langle H_2\mathbf{A}_1, \mathbf{B}_\alpha\rangle = \left(U'_w B_{\alpha 1} + i\omega B_{\alpha 3}\right)_{y=0} \int_{-\infty}^{+\infty} \frac{\rho(\alpha_v)}{i(\alpha_v - \alpha)} \exp\left[i\varepsilon^{-1}\alpha_v x_1\right] d\alpha_v. \tag{3.1}$$

We examine the case of subcritical frequencies of the vibrator oscillations for which all the discrete spectrum waves damp out downstream. To evaluate the integral in (3.1), we close the contour of integration by the arc of a circle in the upper half-plane of the complex α_0 plane. Allowing the radius of the circle to become infinite, and taking into account that $x_1 > 0$, we find

$$\langle H_2\mathbf{A}_1, \mathbf{B}_\alpha\rangle = \left(U'_w B_{\alpha 1} + i\omega B_{\alpha 3}\right)_{y=0} 2\pi\rho(\alpha)\, e^{i\frac{\alpha}{\varepsilon}x_1},$$

where α belongs to the spectrum located in the upper half-plane.

Therefore, the perturbations being excited downstream by a vibrator for $x > x_1 > 0$ are

$$\mathbf{A}(x, y) = \sum_\nu{}' 2\pi\rho(\alpha_\nu)\left(U'_w B_{\alpha_\nu 1} + i\omega B_{\alpha_\nu 3}\right)_{y=0} \mathbf{A}_{\alpha_\nu} e^{i\frac{\alpha_\nu}{\varepsilon}x} + \sum_j{}' \int_0^\infty 2\pi\rho(\alpha_j)\left(U'_w B_{\alpha_j 1} + i\omega B_{\alpha_j 3}\right)_{y=0} \mathbf{A}_{\alpha_j} e^{i\frac{\alpha_j x}{\varepsilon}}\, dk,$$

where $\sum_\nu{}'$, $\sum_j{}'$ denote the summation over the discrete spectrum, and over branches of the continuous spectrum located in the upper half-plane. In particular, we obtain a relationship that agrees with the principal approximation (2.6) for the amplitude of the T–S wave being excited, when the flow is weakly inhomogeneous:

$$\Phi_{TS\Sigma} = 2\pi a\rho(\alpha_{TS})\left(U'_w B_{TS1} + i\omega B_{TS3}\right)_{y=0} \mathbf{A}_{TS} e^{i\frac{\alpha_{TS}x}{\varepsilon}}. \tag{3.2}$$

An awkward analysis executed by the method of mergeable asymptotic expansions within the framework of the model [9] showed that the amplitude (3.2) of the T–S wave agrees identically with the result in [9].

TABLE 1

M	$F\cdot10^6$	Re*	$\alpha_{TS}\cdot10^2$	$(q_m/a)\cdot10^3$	M	$F\cdot10^6$	Re*	$\alpha_{TS}\cdot10^2$	$(q_m/a)\cdot10^3$
0,2	20	1020	7,27	5,79	0,6	20	960	6,45	4,67
—	40	685	8,84	6,46	—	40	645	7,85	5,23
—	60	550	10,04	6,92	—	60	520	8,97	5,65
0,4	20	1010	7,02	5,37	0,8	20	900	5,73	3,81
—	40	665	8,40	5,95	—	40	615	7,12	4,38
—	60	540	9,64	6,42	—	60	490	8,06	4,72

As the frequency of vibrator oscillations increases from sub- to post-critical frequencies, the eigenvalue of the T—S wave α_{TS} goes from the upper half-plane of the complex α plane into the lower (the dashed line in Fig. 2). In this case, a vibrator starting to oscillate at the initial time t = 0 as in the model problem in [6]† must be considered for the correct selection of the contour of integration.

The contribution from the pole $\alpha = \alpha_{TS}$ must be included in the solution for x > 0 by deforming the contour of integration as in Fig. 2. This contour selection rule is in agreement with that postulated in [10]. Let us note that taking account of the main flow not being parallel permits formulating the problem with initial data for $x = x_0$ in large length scales and removes the question of the passage from sub- to post-critical frequencies.

4. It is shown in [11] that the relationships (2.5) and (2.6) do not change for a boundary layer in a compressible gas if it is considered that the first and third components of the vector A for the perturbation amplitude in a compressible gas correspond to perturbations in the velocity x- and y-components.

The relationships (2.6) and (3.2) depend on normalization of the eigenfunctions A_{TS}, B_{TS}. To obtain invariant relations, the factor $S_{TS}\beta/\langle H_2A_{TS}, B_{TS}\rangle$ must be inserted in these expressions, where S_{TS} is the perturbation amplitude of the physical quantity of interest to us, computed along the vector A_{TS}. In the computations S_{TS} equals the maximum magnitude of the perturbation in y for the mass flow rate modulus $q_{TS}(x)$. Then the amplitude of the instability wave excited by a localized vibrator is

$$q_m(x) = aq_{TS}(x)\left|\frac{2\pi\rho(\alpha_{TS})\left(U'_wB_{TS1} + i\omega B_{TS3}\right)_{*,y=0}}{\langle H_2\mathbf{A}_{TS}, \mathbf{B}_{TS}\rangle_*}\right| e^{\mathrm{Im}(F_{TS})}. \quad (4.1)$$

Numerical computations were performed by using (4.1) for a vibrator on a heat-insulated plate around which flows a gas with adiabatic index 1.41, and Prandtl number 0.72. The Mach number in the free stream varied in the range M = 0.2-0.8, the stagnation temperature equaled 310°K, and the viscosity coefficient was computed by the Sutherland formula.

The results of computing the perturbation amplitudes for the mass flow rate $q_m(x_*)$ in a T—S wave excited by a vibrator with $2\pi\rho(\alpha_{v*}) = 1$ localized near the point of loss of stability x_* are given in the table where values of α_{TS} and Re* corresponding to $x = x_*$ are also presented. The characteristic scale X in the computations is equal to the distance between the nose of the plate and the center of the vibrator such that Re = Re*. The data in the table indicate that because of the narrowness of the resonance interaction domain in the wave numbers α_v, the amplitude of a T—S wave generated by a local vibrator is two orders of magnitude less than the corresponding amplitude for the resonance excitation mode on a solitary vibration wave [11].

To compare theory and experiment [12], computations were performed for the r.m.s. pulsations of the x-component of the velocity $u_{TS}(x)$ in an excited T—S wave, corresponding to maximal values in y. The calculations were carried out for Re = 502 at the frequency parameters $F = (50\text{-}120)\cdot10^{-6}$ ($F = 2\pi f\nu_\infty/U_\infty^2$, f is the perturbation frequency in Hz). In conformity with experiment, the shape of the vibrator was given as a trapezoid with lower base l = 28 mm and upper base equal to $2/3l$. Shown in Fig. 3 is the dependence of $u_{TS}(x_*)$ on F for the dimensional pulsation amplitude a = 10 μm. As in the experiment, the amplitude of the excited T—S wave depends weakly on the frequency parameter for the sub- and post-critical frequencies (the critical value is $F \simeq 70\cdot10^{-6}$).

Presented in Fig. 4 is the comparison between theoretical computations (curve 1) and experiment (curve 2), for the vibrator oscillation frequency f = 70 Hz ($F = 120\cdot10^{-6}$). The ab-

†The exact analysis of the problem of a vibrator starting to oscillate at the time t = 0 is executed by E. D. Terent'ev.

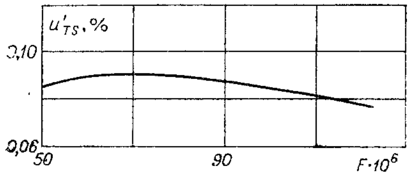

Fig. 3

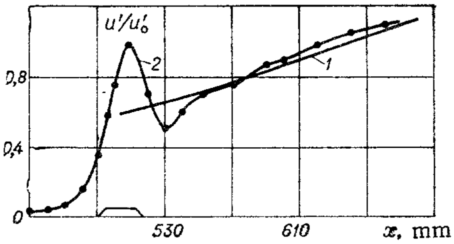

Fig. 4

solute value of the pulsations is referred to the maximal value in x, or u'_0 = 0.13%, achieveable for x ≃ 510 mm [12]; the vibration amplitude is a = 10 μm. In the domain x ⩾ 550 mm where the T—S wave is extracted from the total signal, the theory is in good agreement with experimental data.

It follows from the computations that to excite instability waves with "dangerous" (from the transition viewpoint) frequencies and with initial amplitudes $\sim 10^{-2}$% very small vibrations of the streamlined surface at the same frequency with amplitudes of ∿1 μm are sufficient.

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